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REPORT

CD NO.

50X1-HUM

COUNTRY	USSR	DATE OF INFORMATION	1947
SUBJECT	Scientific - Electromagnetic theory		
HOW PUBLISHED	Monthly periodical	DATE DIST.	1 NOV 1949
WHERE PUBLISHED	Moscow	NO. OF PAGES	6
DATE PUBLISHED	Mar 1948		
LANGUAGE	Russian	SUPPLEMENT TO REPORT NO.	

THIS IS UNEVALUATED INFORMATION

SOURCE Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, Vol XVIII, No 3, 1948.

50X1-HUM

THE THEORY OF THE POINT ELECTRON

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1 Aug 1947

Introduction

As we all know, the laws of classical physics are not applied to the atom. The motion of electrons was studied by the aid of quantum mechanics. It would seem necessary to approach the problem of the structure of elementary particles, especially the electron, exclusively from the viewpoint of quantum laws. But the present quantum theory offers no solution of this difficulty in connection with the nature of the mass of elementary particles. Hence, comparatively recent work has been undertaken anew on the classical theory of elementary particles, in which one of the founders of the quantum mechanics, Dirac /1/, took part.

The possibility of describing some properties of elementary particles by the aid of the classical theory is obviously not an accident. As a rule, quantum laws develop into classical laws when de Broglie's wave length h/mv (h is Planck's constant and mv is the momentum of the particles) is reduced to zero. In particular, the electromagnetic mass of an electron $m^{el} \sim e^2/r_0c^2$ (r_0 is the radius of the electron) does not depend on Planck's constant h, for which reason it is perfectly natural to study this central problem by the classical theory.

Let us emphatically state that elimination of the difficulties connected with electrostatic energy (linear proper energy) can proceed parallel with both the classical and the quantum theories. We remain face to face with specific quantum infinities of the proper energy of a point charge bound to fluctuations in the transverse field (transverse proper energy) and not included in the classical theory /2/.

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It is possible that in future theories about elementary particles applicability of both classical and quantum laws will be limited to a certain minimum distance a to be introduced, for instance, in the theory of quantum space [3]; but if these difficulties are solved within the framework of contemporary quantum theory, we must expect in every case that the magnitudes found which are not dependent on Planck's constant must coincide with the classical limits. Furthermore, the classical standards have considerable heuristic power, often pointing the way to the construction of a more exact quantum theory.

The problem of the nature of the electron mass is basic in the theory of elementary particles. Up to the present, we have not known whether the electron mass originates from the electromagnetic energy of a charged particle (field hypothesis) or whether it is necessary to introduce the mass in the basic equation of motion as some constant magnitude (nonfield hypothesis).

Basic Equations

The equation of motion of a single electron in an external electromagnetic field takes the form:

$$m\ddot{x}^\mu = F_i^\mu + F^\mu, \tag{1}$$

where m is the rest mass and x^μ is the coordinates of the electron (x, y, z, ict), depending on the proper time $s(\dot{x} = dx/ds)$; F_i^μ and F^μ are respectively for forces acting on the electron in the external and internal fields.

The internal force is connected with the electromagnetic field by the relation

$$F_i^\mu = (e/c)\dot{x}^\nu H_i^{\mu\nu} \tag{2}$$

where e is the electron charge. But the force of self-action for an electron of radius r_0 equals

$$F^\mu = -m^{el}\ddot{x}^\mu + (2e^2/3c^3)(\dddot{x}^\mu - c^{-2}\dot{x}^\mu \ddot{x}\ddot{x}). \tag{3}$$

Here, $m^{el} \sim e^2/c^2 r_0$, and the discarded terms are of the order r_0. Expression (3) represents the expansion of F^μ in terms of r_0/λ; it is the relationship between electron radius r_0 and wave-length λ of oscillation of the electron.

Recently, Dirac offered a classical theory of the point electron ($r_0 \to 0$), in the structure of which the following weaknesses appeared:

a. If the radius of the electron tends toward zero ($r_0 \to 0$), the field mass m^{el} becomes infinite. To eliminate this difficulty, some methods have been suggested: the theory of λ-processes, compensation for the infinite electromagnetic mass of an electron by another nonfield mass with a negative value, etc.

It seems to us that the most rational method of eliminating an infinite field mass is to introduce a second non-Maxwell field, acting only upon the electrons producing it [4]. So that a non-Maxwell field may not have radiations in the form of electromagnetic waves, it is necessary in calculating it to take the half-sum of the retardational and advancing potentials. In this case, for the second force of self-action we shall have:

$$F^{\mu} = m^{el}\ddot{x}^{\mu}. \qquad (4)$$

Then the equations of motion of a point electron will take the form first pointed out by Dirac:

$$m\ddot{x}^{\mu} = (e/c)\dot{x}^{\nu}H_{i}^{\mu\nu} + (2e^2/3c^3)(\dddot{x}^{\mu} - c^{-2}\dot{x}^{\mu}\ddot{x}\cdot\ddot{x}) \qquad (5)$$

Let us notice that the variation in mass with velocity of a moving electron is completely identical for both field and nonfield masses. Therefore, experiments made along these lines cannot solve the problem concerning the origin of mass.

Equations of motion, however, for field and nonfield masses will differ in rapidly fluctuation fields ($\lambda > r_0$). For a point electron, equation (5) is employed for wave lengths as small as desired, while for an electron possessing electromagnetic mass (see relation [3]), this equation is only approximate and is admissible for oscillations of electrons whose wave length λ exceeds the electron radius. Finally, it has not been possible with the Maxwell-Lorentz theory, to introduce by the relativistic invariant method a length which plays the part of an electron's radius. Consequently, if we really wish to remain within the framework of the field mass theory, we must make new relativistic invariant theories, among which we may note first of all, Born's nonlinear theory or the Bopp-Podol'skiy field theory with higher order of derivatives.

b. The second difficulty is connected with solving Dirac's equation which, beside acceleration $\ddot{x}^{\mu}$, includes still another derivative of the acceleration $\dddot{x}^{\mu}$. A detailed analysis by Belousov and Markov [5] proved that equation (5) is not a correct solution if at the initial moment the initial acceleration is also assigned in addition to coordinates and velocity. However, our research [4] demonstrates that equation (5) is a correct solution if we require the electron, for $s = \infty$, to move in accordance with inertia; that is, besides initial coordinates and velocity, which can be assigned for $s = -\infty$, a finite acceleration reducing to zero at $s = \infty$, must also be assigned [s = time].

In particular for rectilinear motion taking place outside the electromagnetic field E, we found that:

$$\dot{x} = c\,\mathrm{sh}\,q, \quad ct = c\,\mathrm{ch}\,q, \qquad (6)$$
$$q(s) = q^{r}(s) + q^{a}(s).$$

Here, q^r represents the retardation solution

$$q^{r}(s) = (e/mc)\int_{-\infty}^{s} E(s')\,ds', \qquad (7)$$

since the external field is taken at the moment of time s' just prior to time s, and q^a is the advance or leading solution:

$$q^{a}(s) = \frac{e}{mc}\int_{s}^{\infty} E(s')\,e^{(s-s')/s_0}\,ds'; \qquad (8)$$

50X1-HUM

and

$$cs_0 = (2/3)r_0 = 2e^2/3mc^2. \tag{9}$$

The solution connected with advance action is very interesting. It is as if the electron were distributed along the time axis; consequently, after s_0 seconds it begins to take on the appearance of the external field. After this time the electromagnetic wave is able to traverse the distance:

$$cs_0 = 2r_0/3,$$

which we take as the radius of the electron.

Thus, the field theory introduces the electron radius r_0, and the electromagnetic mass is expressed by means of this radius. In our case, however, the nonfield mass of an electron was introduced which automatically leads to the idea of the radius of an electron.

New additional conditions (reduction of acceleration to 0 at $s = \infty$) influence the formulation of the basic (variantive) principles of classical mechanics.

As is well-known, there are two methods of approaching a variantive problem in classical mechanics. In the Lagrangian method, in which a Lagrangian function is assigned for any time interval of a moving material point, we have as a function of action:

$$S = \int_{-\infty}^{+\infty} L\, ds' \tag{10}$$

In this case we must assign variations at the limits of integration that is, at $s = \pm\infty$. In the other method, the Hamilton-Jacobi method, the action function takes the form:

$$S = \int_{-\infty}^{s} L\, ds', \tag{11}$$

that is, the Lagrangian L is assigned only until the sought for moment of time s. The Hamilton-Jacobi method permits solutions only with initial conditions.

In classical mechanics Lagrangian formulas are considered in which both methods are absolutely identical; that is, when it is sufficient for the solution of the whole problem to assign only initial conditions (the coordinates and velocity). A point electron's equation of motion can be solved only when the limits besides the initial conditions are assigned; therefore, the variantive problem of the motion of a point electron can be formulated only by the aid of the Lagrangian method (see, for example, [4]).

Motion of Two Connected Electrons

The simplest example of electron motion in an external field is the motion of two electrons between which electromagnetic (external) forces are acting. If we denote the magnitudes referring to the first and second electrons respectively by indices 1 and 2, we shall have the following equation describing the first electron's motion:

$$m\ddot{x}_1^\mu = (e/c)\dot{x}_{1\nu} H_2^{\mu\nu} + (2e^2/3c^3)(\dddot{x}_1^\mu - c^{-2}\dot{x}_1^\mu \ddot{x}_1 \cdot \ddot{x}_1). \qquad (12)$$

Here, the field produced by the second electron (let us take the retardation solution) will be determined from the equation:

$$H_2^{\mu\nu} = -\frac{e}{2}\,\frac{1+(t_1-t_2)/|t_1-t_2|}{\dot{x}_2(x_1-x_2)}\,\frac{d}{ds_2}\,\frac{\dot{x}_2^\nu(x_1^\mu-x_2^\mu)-\dot{x}_2^\mu(x_1^\nu-x_2^\nu)}{\dot{x}_2\cdot(x_1-x_2)} \qquad (13)$$

The coordinates of the first electron x_1^μ depend on its own time s_1, and the second on s_2. Both these times are connected by the relation

$$(x_1-x_2)\cdot(x_1-x_2)=0. \qquad (14)$$

To obtain the equation of motion for the second electron we must interchange the indices 1 and 2 in the last equations. In the general case of a system of equations describing the motion of two electrons it is rather difficult to find a solution, especially if the variables x_1 and x_2 are considered to depend upon different arguments.

Moreover, we want to analyze the coupled oscillation of two electrons, that is, the motion in which the electrons oscillate with a wave length that often exceeds the distance R between them. For the sake of simplicity, we shall limit ourselves to nonrelativistic uniform motion. Assuming in (12) that $s_1 = t$, $s_2 = t'$, we shall then have $x_1 = x_1(t)$, $x_2 = x_2(t')$.

Disregarding terms of the order $(x_1/c)^2$ and so forth in equations (12) and (13), we shall find that the following equations determine the oscillations of both electrons:

$$m\ddot{x}_1 = F_{12} + (2e^2/3c^3)\dddot{x}_1,$$
$$m\ddot{x}_2 = F_{21} + (2e^2/3c^3)\dddot{x}_2. \qquad (15)$$

where F_{12} and F_{21} are forces of interaction. In addition, the right parts of the latter equations must imply an external force communicating oscillatory motion to the electrons.

In our case of rectilinear motion, we have:

$$F_{12} = -e^2/R'^2 + 2e^2\dot{x}_2(t')/cR'^2, \qquad (16)$$

where $R' = |x_2(t') - x_1(t)|$, $t' = t - R/c$, $R = x_2(t) - x_1(t)$, and the last equation is written on the assumption that $x_2(t) > x_1(t)$.

Expanding F_{12} in the variable R/c and discarding terms proportional to R, we shall obtain

$$F_{12} = -\frac{e^2}{R^2} - \frac{e^2\ddot{x}_2(t)}{c^2R} + \frac{2e^2\dddot{x}_2(t)}{3c^3}. \qquad (17)$$

It stands to reason that the indicated expansion possesses significance when

$$(R/c)(\ddot{x}_2/\dot{x}_2) \ll 1$$

or in the case of harmonic oscillations

$$R/\lambda \ll 1. \tag{18}$$

By a similar method it is easy to obtain

$$F_{21} = \frac{e^2}{R^2} - \frac{e^2 \ddot{x}_1(t)}{c^2 R} + \frac{2e^2 \dddot{x}_1(t)}{3c^3}. \tag{19}$$

Substituting the last expression in (15), let us find the following equation for the oscillation of the centroid $x = (x_1 + x_2)/2$ of both electrons:

$$2m\ddot{x} = -\frac{(2e)^2}{2c^2R}\ddot{x} + \frac{2(2e)^2}{3c^3}\dddot{x}. \tag{20}$$

Hence, we see that the concept of electromagnetic mass preserves its significance even when two or more reacting particles are present. With this electromagnetic mass, there is added to the total mass of particles

$$m^{el} = (2e)^2/2c^2R, \tag{21}$$

When there are repelling forces, this mass has a positive value; and when there are forces of attraction, this mass is a negative quantity. It means that for forces of friction and for the total energy of radiation the law of additivity does not hold true, inasmuch as the force of friction is proportional to the square of the charge.

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